Laboratory Project

CAS Logistic Sequences

A sequence that arises in ecology as a model for population growth is defined by the **logistic difference equation**

$$p_{n+1} = kp_n(1 - p_n)$$

where p_n measures the size of the population of the nth generation of a single species To keep the numbers manageable, p_n is a fraction of the maximal size of the population, so $0 \leq p_n \leq 1$. Notice that the form of this equation is similar to the logistic differential equation in Section 7.5 The discrete model—with sequences instead of continuous functions—is preferable for modeling insect populations, where mating and death occur in a periodic fashion.

An ecologist is interested in predicting the size of the population as time goes on, and asks these questions: Will it stabilize at a limiting value? Will it change in a cyclical fashion? Or will it exhibit random behavior?

Write a program to compute the first n terms of this sequence starting with an initial population p_0, where $0 < p_0 < 1$ Use this program to do the following

1. Calculate 20 or 30 terms of the sequence for $p_0 = \frac{1}{2}$ and for two values of k such that $1 < k < 3$ Graph the sequences Do they appear to converge? Repeat for a different value of p_0 between 0 and 1 Does the limit depend on the choice of p_0? Does it depend on the choice of k?
2. Calculate terms of the sequence for a value of k between 3 and 3.4 and plot them. What do you notice about the behavior of the terms?
3. Experiment with values of k between 3.4 and 3.5. What happens to the terms?
4. For values of k between 3.6 and 4, compute and plot at least 100 terms and comment on the behavior of the sequence. What happens if you change p_0 by 0.001? This type of behavior is called *chaotic* and is exhibited by insect populations under certain conditions