39. World oil consumption was 75 747 million barrels per day in 2002 and is increasing by about 0 3% per year [3] Let c_n be daily world oil consumption n years after 2002

(a) Find a formula for c_n.
(b) Find and interpret $c_n - c_{n-1}$.
(c) What does the sum $\sum_{n=1}^{18} c_n$ represent? (You do not need to compute this sum)

40. (a) Let s_n be the number of ancestors a person has n generations ago What is s_1? s_2? Find a formula for s_n
(b) For which n is s_n greater than 6 billion, the current world population? What does this tell you about your ancestors?

41. For $0 \leq n \leq 10$, find a formula for p_n, the payment in year n on a loan of \$100,000 Interest is 5% per year, compounded annually, and payments are made at the end of each year for ten years Each payment is \$10,000 plus the interest on the amount of money outstanding

42. Baby formula can contain bacteria which double in number every half hour at room temperature and every 10 hours in the refrigerator [4] Suppose there are B_0 bacteria initially

(a) Write a formula for
  (i) R_n, the number of bacteria n hours later if the baby formula is kept at room temperature
  (ii) F_n, the number of bacteria n hours later if the baby formula is kept in the refrigerator
  (iii) Y_n, the ratio of the number of bacteria at room temperature to the number of bacteria in the refrigerator
(b) How many hours does it take before there are a million times as many bacteria in baby formula kept at room temperature as in baby formula kept in the refrigerator?

43. You are deciding whether to buy a new or a two-year-old car (of the same make) based on which will have cost you less when you resell it at the end of three years Your cost consists of two parts: the loss in value of the car and the repairs A new car costs \$20,000 and loses 12% of its value each year Repairs are \$400 the first year and increase by 18% each subsequent year

(a) For a new car, find the first three terms of the sequence d_n giving the depreciation (loss of value) in dollars in year n Give a formula for d_n
(b) Find the first three terms of the sequence r_n the repair cost in dollars for a new car in year n Give a formula for r_n
(c) Find the total cost of owning a new car for three years
(d) Find the total cost of owning the two-year-old car for three years Which should you buy?

44. Write a definition for $\lim_{n\to\infty} s_n = L$ similar to the ϵ, δ definition for $\lim_{x\to a} f(x) = L$ in Section 1 8 Instead of δ, you will need N, a value of n

45. The sequence s_n is increasing, the sequence t_n converges, and $s_n \leq t_n$ for all n Show that s_n converges

In Exercises 46–51, find a recursive definition for the sequence

46. $1, 3, 5, 7, 9, \ldots$

47. $2, 4, 6, 8, 10, \ldots$

48. $3, 5, 9, 17, 33, \ldots$

49. $1, 5, 14, 30, 55, \ldots$

50. $1, 3, 6, 10, 15, \ldots$

51. $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \ldots$

In Problems 52–54, show that the sequence s_n satisfies the recurrence relation

52. $s_n = 3n - 2$
$s_n = s_{n-1} + 3$ for $n > 1$ and $s_1 = 1$

53. $s_n = n(n+1)/2$
$s_n = s_{n-1} + n$ for $n > 1$ and $s_1 = 1$

54. $s_n = 2n^2 - n$
$s_n = s_{n-1} + 4n - 3$ for $n > 1$ and $s_1 = 1$

55. (a) Cans are stacked in a triangle on a shelf The bottom row contains k cans, the row above contains one can fewer, and so on How many rows are there? Find a_n, the number of cans in the n^{th} row from the top, $1 \leq n \leq k$
(b) Let T_n be the total number of cans in the top n rows Find a recurrence relation for T_n in terms of T_{n-1}
(c) Show that $T_n = \frac{1}{2}n(n+1)$ satisfies the recurrence relation

56. The Fibonacci sequence first studied by the thirteenth century Italian mathematician Leonardo di Pisa, also known as Fibonacci, is defined recursively by

$$F_n = F_{n-1} + F_{n-2} \text{ for } n > 2 \text{ and } F_1 = 1, F_2 = 1$$

The Fibonacci sequence occurs in many branches of mathematics and can be found in patterns of plant growth (phyllotaxis)

(a) Find the first 12 terms.
(b) Show that the sequence of successive ratios F_{n+1}/F_n appears to converge to a number r satisfying the equation $r^2 = r + 1$ (The number r was known as the golden ratio to the ancient Greeks)
(c) Let r satisfy $r^2 = r + 1$ Show that the sequence $s_n = Ar^n$, where A is constant, satisfies the Fibonacci equation $s_n = s_{n-1} + s_{n-2}$ for $n > 2$

[4] Iverson C and Forsythe F, reported in "Baby Food Could Trigger Meningitis" www.newscientist.com June 3, 2004