Since the sequence $\{a_n\}$ is increasing and bounded, the Monotonic Sequence Theorem guarantees that it has a limit The theorem doesn't tell us what the value of the limit is But now that we know $L = \lim_{n\to\infty} a_n$ exists, we can use the given recurrence relation to write

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \tfrac{1}{2}(a_n + 6) = \tfrac{1}{2}\Big(\lim_{n\to\infty} a_n + 6\Big) = \tfrac{1}{2}(L + 6)$$

Since $a_n \to L$, it follows that $a_{n+1} \to L$ too (as $n \to \infty$, $n + 1 \to \infty$ also) So we have

$$L = \tfrac{1}{2}(L + 6)$$

Solving this equation for L, we get $L = 6$, as predicted.

8.1 Exercises

1. (a) What is a sequence?
   (b) What does it mean to say that $\lim_{n\to\infty} a_n = 8$?
   (c) What does it mean to say that $\lim_{n\to\infty} a_n = \infty$?

2. (a) What is a convergent sequence? Give two examples
   (b) What is a divergent sequence? Give two examples

3. List the first six terms of the sequence defined by

$$a_n = \frac{n}{2n+1}$$

   Does the sequence appear to have a limit? If so, find it

4. List the first eight terms of the sequence $\{\sin(n\pi/2)\}$. Does this sequence appear to have a limit? If so, find it. If not, explain why

5–8 ■ Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

5. $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\}$ 6. $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots\}$

7. $\{2, 7, 12, 17, \dots\}$ 8. $\{0, 2, 0, 2, 0, 2, \dots\}$

9–26 ■ Determine whether the sequence converges or diverges. If it converges, find the limit.

9. $a_n = n(n-1)$ 10. $a_n = \dfrac{n+1}{3n-1}$

11. $a_n = \dfrac{3+5n^2}{n+n^2}$ 12. $a_n = \dfrac{\sqrt{n}}{1+\sqrt{n}}$

13. $a_n = \dfrac{2^n}{3^{n+1}}$ 14. $a_n = \dfrac{n}{1+\sqrt{n}}$

15. $a_n = \dfrac{(-1)^{n-1}n}{n^2+1}$ 16. $\{\arctan 2n\}$

17. $a_n = 2 + \cos n\pi$ 18. $a_n = \dfrac{n\cos n}{n^2+1}$

19. $\left\{\dfrac{\ln(n^2)}{n}\right\}$ 20. $\{(-1)^n \sin(1/n)\}$

21. $\{\sqrt{n+2} - \sqrt{n}\}$ 22. $\left\{\dfrac{\ln(2+e^n)}{3n}\right\}$

23. $a_n = n2^{-n}$ 24. $a_n = \ln(n+1) - \ln n$

25. $a_n = \dfrac{\cos^2 n}{2^n}$ 26. $a_n = \dfrac{(-3)^n}{n!}$

27–32 ■ Use a graph of the sequence to decide whether the sequence is convergent or divergent. If the sequence is convergent, guess the value of the limit from the graph and then prove your guess (See the margin note on page 568 for advice on graphing sequences.)

27. $a_n = (-1)^n \dfrac{n+1}{n}$ 28. $a_n = 2 + (-2/\pi)^n$

29. $\left\{\arctan\left(\dfrac{2n}{2n+1}\right)\right\}$ 30. $\left\{\dfrac{\sin n}{\sqrt{n}}\right\}$

31. $a_n = \dfrac{n^3}{n!}$

32. $a_n = \dfrac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{(2n)^n}$

33. If \$1000 is invested at 6% interest, compounded annually, then after n years the investment is worth $a_n = 1000(1.06)^n$ dollars.
   (a) Find the first five terms of the sequence $\{a_n\}$
   (b) Is the sequence convergent or divergent? Explain